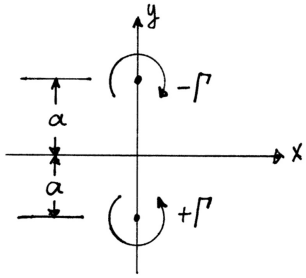
Fundamentals of Fluid Mechanics A
Assignment 10 — Potential Vortex

Instructions

Write your solutions in single column format, with one statement following another vertically. Write your solutions neatly so that they are easy to read and verify. Don't write one line with two equal signs. Highlight your answers using a box. Failure to do this will result in a lower score and fewer comments on my part.

Question #1

We can construct another fundamental solution to Laplace's equation, known as the dipole, using two potential vortices as follows:



Assume the circulation Γ, is such that the dipole strength, $\mathcal{D}_p$, is given by:

$$\mathcal{D}_p = \lim_{a \to 0} 2a\Gamma$$

Do the following:

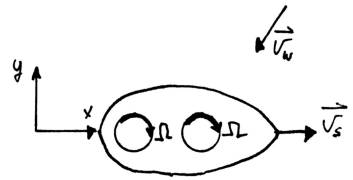
(a) Determine the velocity potential.

(b) Determine the stream function.

(c) Compare your results with those of a doublet.

Question #2

The Flettner-Rotor ship was designed with two rotating cylinders of height H acting as sails:



The cylinders have diameter D and rotate with angular velocity Ω. The ship moves with velocity $\vec{v}_s = U\vec{i}$, and the wind velocity is $\vec{v}_w = -W(\cos(\alpha)\vec{i} + \sin(\alpha)\vec{j})$, and the air density is ρ. Do the following:

(a) Estimate the force on the ship as a function of Ω, D, H, U, W, α, and ρ.

(b) If the side force vanishes, what is the thrust?

Question #3

A simplified model of a hurricane approximates the flow as a rigid-body rotation in the inner core and a potential vortex outside of the core. That is, the circumferential velocity, $\vec{v}_\theta$, is:

$$\vec{v}_\theta = \begin{cases} \dfrac{\Gamma r}{2\pi R^2} & \text{for } r \leq R \\ \dfrac{\Gamma}{2\pi r} & \text{for } r \geq R \end{cases}$$

where r is the radial distance, R is the core radius, and Γ is the circulation. Do the following:

(a) Plot qualitatively $\vec{v}_\theta$ as a function of the radius r.

(b) Determine the velocity potential.

(c) Determine the pressure for $P \to P_\infty$ as $r \to \infty$.

(d) What is the minimum pressure if $\rho = 1.206$ kg/m^3, $P_\infty = 101300$ Pa, and the peak velocity is 322 km/hr?

Answers

1. (a) $\mathcal{D}_p \cos(\theta)/(2\pi r)$
2. $\pi\rho\Omega D^2 H(W\sin(\alpha)\vec{i} - (U + W\cos(\alpha))\vec{j})$, $\pi\rho\Omega D^2 H\sqrt{W^2 - U^2}$
3. $P_\infty - 0.5\rho\Gamma^2/(2\pi r)^2$, 96.5 kPa.

Due on November 30th at 11:00. Do all 3 problems.