Fundamentals of Fluid Mechanics B Assignment 1 — Mean Free Path, Viscosity, and Viscous Diffusion

Instructions

Write your solutions in single column format, with one statement following another vertically. Write your solutions neatly so that they are easy to read and verify. Don't write one line with two equal signs. Highlight your answers using a box. Failure to do this will result in a lower score and fewer comments on my part.

Question #1

Starting from the conservation of momentum principle, derive Newton's law for a system with mass aggregation as follows:

$$\sum \vec{F} = m\frac{d\vec{v}}{dt} - \sum_k \dot{m}_k \vec{v}_{\text{REL},k}$$

Do the proof with 3 small masses dm_1, dm_2, dm_3 being added to one large mass m.

Question #2

Starting from the Newton's law for a system with mass aggregation:

$$\sum \vec{F} = m\frac{d\vec{v}}{dt} - \sum_k \dot{m}_k \vec{v}_{\text{REL},k}$$

Derive the x momentum equation:

$$\rho\frac{du}{dt} = -\frac{\partial P}{\partial x} - \frac{\partial f_{ux}}{\partial x} - \frac{\partial f_{uy}}{\partial y} - \frac{\partial f_{uz}}{\partial z}$$

Define clearly the viscous fluxes f_{ux}, f_{uy}, etc.

Question #3

Starting from the definition of the viscous flux f_{ux}, prove that f_{ux} is equal to:

$$f_{ux} = -\mu\frac{\partial u}{\partial x}$$

with the viscosity equal to:

$$\mu \equiv \frac{2}{3}\sqrt{\frac{k_{\text{B}}Tm}{\pi\sigma^2}}$$

Explain assumptions and limitations if any. In deriving the latter, you can use the following relationship for the mean free path in a gas:

$$\lambda = \frac{1}{\sqrt{2}N\sigma}$$

Question #4

Prove that the mean free path in a gas taking into consideration the relative velocity of the molecules corresponds to:

$$\lambda = \frac{1}{\sqrt{2}N\sigma}$$

Note: you need to prove everything in this equation, including the $\sqrt{2}$ on the denominator.

Due on Thursday February 4th at 11:00. Do Questions 1-3. Question #4 is optional but recommended.