

Fundamentals of Fluid Mechanics B

Assignment 2 — Navier Stresses and Stokes Hypothesis

Question #1

Recall the normal and shear strain rates:

$$S_{xx} = \frac{\partial u}{\partial x}$$

$$S_{yy} = \frac{\partial v}{\partial y}$$

and

$$S_{xy} = S_{yx} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

Do the following:

- Prove that S_{xx} and S_{yy} become zero for pure translation and no volume distortion.
- Prove that S_{xy} becomes zero for pure rotation without angular distortion.
Hint: start with the angles of distortion with respect to the x and y axes.

Question #2

Starting from the angular distortion and volume expansion of a fluid element, show that the shear stresses for a fluid with a linear stress-strain relationship become:

$$\tau_{xx} = a \frac{\partial u}{\partial x}$$

$$\tau_{yy} = a \frac{\partial v}{\partial y}$$

$$\tau_{zz} = a \frac{\partial w}{\partial z}$$

and

$$\tau_{xy} = \tau_{yx} = \frac{a}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\tau_{xz} = \tau_{zx} = \frac{a}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

$$\tau_{yz} = \tau_{zy} = \frac{a}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

where a is a constant of proportionality.

Question #3

Starting from the Navier stresses obtained in the previous section and applying Stokes hypothesis, show that the Navier-Stokes normal shear stresses become:

$$\begin{aligned}\tau_{xx} &= \frac{a}{2} \left(\frac{4}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial w}{\partial z} \right) \\ \tau_{yy} &= \frac{a}{2} \left(\frac{4}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial w}{\partial z} \right) \\ \tau_{zz} &= \frac{a}{2} \left(\frac{4}{3} \frac{\partial w}{\partial z} - \frac{2}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} \right)\end{aligned}$$

Question #4

Starting from the non-constant-density and non-constant-viscosity Navier-Stokes equations and the mass conservation transport equation, show that the Navier-Stokes constant-density y -momentum equation corresponds to:

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial P}{\partial y} + \frac{a}{2} \frac{\partial^2 v}{\partial x^2} + \frac{a}{2} \frac{\partial^2 v}{\partial y^2} + \frac{a}{2} \frac{\partial^2 v}{\partial z^2}$$

Also explain why

$$a = 2\mu$$

Question #5

Prove that the shear strain rate in yz is equal to:

$$S_{yz} = \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

Justify all your steps clearly including the factor $\frac{1}{2}$.

Due on Thursday February 11th at 11:00. Do Questions #1, #4, and #5 only.