Fundamentals of Fluid Mechanics B Assignment 2 — Navier Stresses and Stokes Hypothesis

Question #1

Recall the normal and shear strain rates:

$$
\begin{aligned}
S_{xx} &= \frac{\partial u}{\partial x} \\
S_{yy} &= \frac{\partial v}{\partial y}
\end{aligned}
$$

and

$$
S_{xy} = S_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)
$$

Do the following:

(a) Prove that S_{xx} and S_{yy} become zero for pure translation and no volume distortion.

(b) Prove that S_{xy} becomes zero for pure rotation without angular distortion. Hint: start with the angles of distortion with respect to the x and y axes.

Question #2

Starting from the angular distortion and volume expansion of a fluid element, show that the shear stresses for a fluid with a linear stress-strain relationship become:

$$
\begin{aligned}
\tau_{xx} &= a\frac{\partial u}{\partial x} \\
\tau_{yy} &= a\frac{\partial v}{\partial y} \\
\tau_{zz} &= a\frac{\partial w}{\partial z}
\end{aligned}
$$

and

$$
\begin{aligned}
\tau_{xy} &= \tau_{yx} = \frac{a}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\
\tau_{xz} &= \tau_{zx} = \frac{a}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\
\tau_{yz} &= \tau_{zy} = \frac{a}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)
\end{aligned}
$$

where a is a constant of proportionality.

Question #3

Starting from the Navier stresses obtained in the previous section and applying Stokes hypothesis, show that the Navier-Stokes normal shear stresses become:

$$\begin{aligned}
\tau_{xx} &= \frac{a}{2}\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right) \\
\tau_{yy} &= \frac{a}{2}\left(\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial w}{\partial z}\right) \\
\tau_{zz} &= \frac{a}{2}\left(\frac{4}{3}\frac{\partial w}{\partial z} - \frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)
\end{aligned}$$

Question #4

Starting from the non-constant-density and non-constant-viscosity Navier-Stokes equations and the mass conservation transport equation and the strain rates listed in the tables, show that the Navier-Stokes constant-density y-momentum equation corresponds to:

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + \frac{a}{2}\frac{\partial^2 v}{\partial x^2} + \frac{a}{2}\frac{\partial^2 v}{\partial y^2} + \frac{a}{2}\frac{\partial^2 v}{\partial z^2}$$

Also explain why

$a = 2\mu$

Question #5

Prove that the shear strain rate in yz is equal to:

$$S_{yz} = \lim_{\Delta t \to 0} \frac{1}{2\Delta t}\left(\frac{y_{\mathrm{A}'} - y_{\mathrm{O}'}}{\xi} + \frac{z_{\mathrm{B}'} - z_{\mathrm{O}'}}{\eta}\right)$$

Make sure to explain clearly the origin of all the terms including the factor $\frac{1}{2}$. Then prove that the latter becomes:

$$S_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

Do Questions #1 and #5 only. Due on Tuesday February 6th at 11:00.