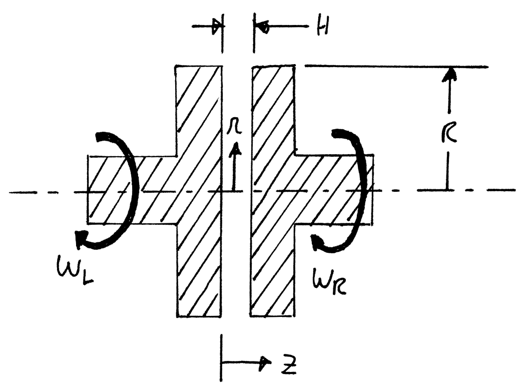
Fundamentals of Fluid Mechanics B Assignment 4 — Nearly Parallel Viscous Flow

Question #1

Two equally big circular plates rotate very close to each other in a viscous fluid as follows:



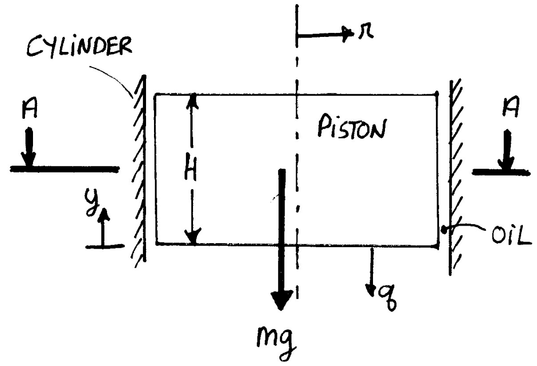
One of the plates is driven by a constant power $\mathcal{P}_\mathrm{L}$ and at a constant angular speed ω_L. The other plate is braked with a power $\mathcal{P}_\mathrm{R}$. For a plate radius R much larger than the distance between the plates H, do the following:

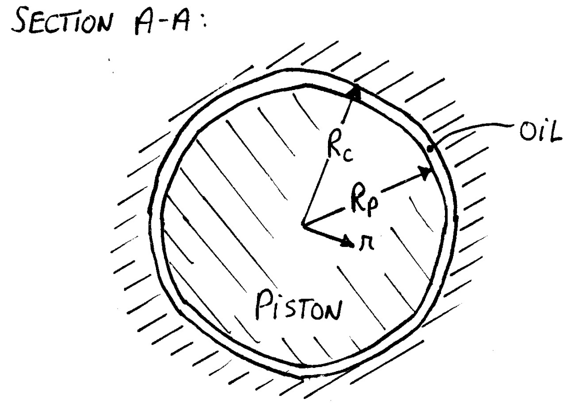
(a) Determine the angular speed ω_R as a function of ω_L and if the breaking power $\mathcal{P}_\mathrm{R} = \frac{1}{2}\mathcal{P}_\mathrm{L}$

(b) If $\omega_\mathrm{R} = 0$, $\mu = 10^{-2}$ kg/ms, $\rho = 800$ kg/m^3, $R = 0.1$ m, and $H = 3$ mm, determine the amount of power $\mathcal{P}_\mathrm{L}$ needed to sustain $\omega_\mathrm{L} = 3000$ rpm.

Question #2

Consider the following piston-cylinder assembly:





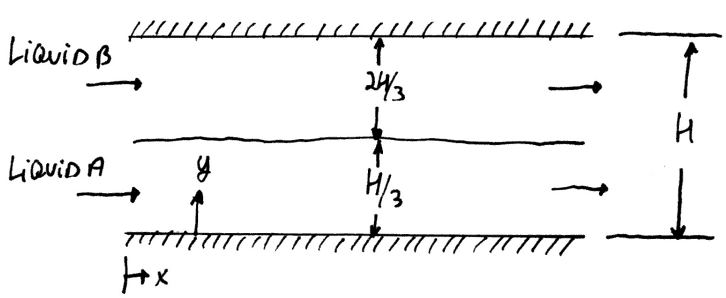
In the latter, the cylinder is fixed while the piston is allowed to move and is subject to a gravity force mg. Knowing that the gravitational acceleration is of $g = 9.8\ \text{m/s}^2$, that the radius of the piston and of the cylinder are of $R_\text{p} = 10$ cm and $R_\text{c} = 10.3$ cm, respectively, that the height of the piston is of $H = 5$ cm, that the density of the piston is of $\rho_\text{p} = 2000\ \text{kg/m}^3$, and that the oil viscosity and density are of $\mu_\text{oil} = 0.5$ kg/ms and $\rho_\text{oil} = 800\ \text{kg/m}^3$, do the following:

(a) Find the force acting on the piston in the positive y direction due to viscous effects as a function of the piston speed q. For simplicity, you can assume that $R_\text{c} - R_\text{p} \ll R_\text{c}$.

(b) Using the expression derived in (a), find the maximum speed q that the piston would get if it is allowed to fall freely assuming negligible drag on its top and bottom surfaces.

Question #3

Consider fully-developed flow between two plates as follows:



Two fluids are flowing between the two plates and do not mix with one another. Focus on the fully-developed region and assume that the density and viscosity of each fluid is known. Start from the mass and momentum conservation equations and do the following:

(a) Find the pressure distribution along y.

(b) Find the velocity distribution in fluid A.

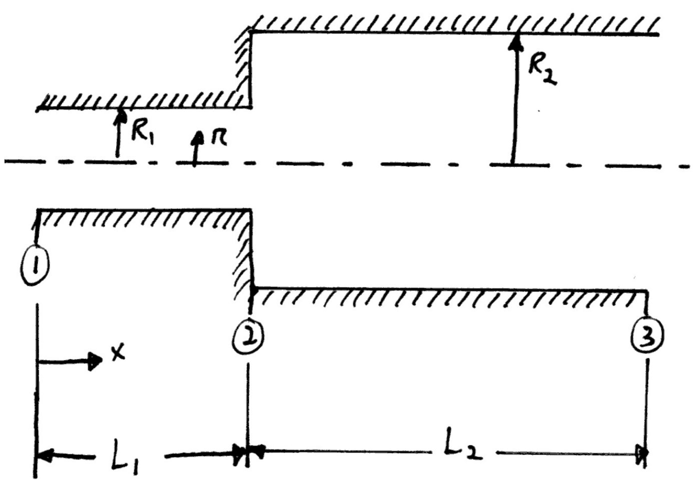
(c) Find the velocity distribution in fluid B.

(d) Find the shear stress on each wall and add them together to find τ_w.

(e) Find the mass flow rate of fluid A within the duct.

Question #4

Consider a fluid flow in a pipe system as follows:



Given the pipe lengths L_1, L_2, and the pipe radii R_1 and R_2, as well as the fluid density ρ and viscosity μ, derive an expression (starting from the equations listed in the tables) for the pressure difference $P_1 - P_3$ needed to entrain a mass flow rate $\dot{m}$. Outline clearly your assumptions.

Question #5

Recall that for Poiseuille flow between two plates, we obtained:

$$\frac{\dot{m}}{W} = -\frac{\rho H^3}{12\mu}\frac{\partial P}{\partial x}$$

$$v = \frac{y}{2\mu}\frac{\partial P}{\partial x}(y - H)i$$

where W is the width of the plates (along z) and H is the distance between the two plates (along y). Do the following:

(a) Find the wall shear stress τ_w on each plate due to the fluid friction.

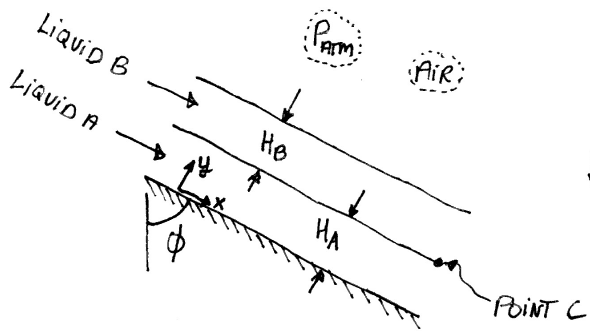
(b) Derive an expression for the Darcy friction factor function of Reynolds number. Clearly define your Reynolds number.

(c) Write down the hydraulic diameter for this problem.

(d) Rewrite your Reynolds number and friction factor in terms of the hydraulic diameter.

Question #6

Consider two fluid layers flowing along a plane as follows:



Given the plane inclination ϕ, the gravitational acceleration g, as well as the fluid properties $\rho_{\rm A}$, $\mu_{\rm A}$, $\rho_{\rm B}$, $\mu_{\rm B}$, and starting from the mass and momentum transport equations, do the following:

(a) Knowing that the speed of the flow at point C is q_C, derive an expression for the velocity within fluid A and fluid B as a function of q_C, and x, y, $H_{\rm A}$, $H_{\rm B}$, g, ϕ.

(b) Derive an expression for $H_{\rm B}$ as a function of $H_{\rm A}$, $q_{\rm C}$, g, ϕ, and the fluid properties $\rho_{\rm A}$, $\rho_{\rm B}$, $\mu_{\rm A}$, $\mu_{\rm B}$.

Answers

1. $\omega_{\rm L}/2$, 51.7 W.
2. 6.02 m/s.
3. $P = P(x,y)$, $u_{\rm A} = \frac{1}{\mu_{\rm A}}\frac{\partial P}{\partial x}\left(\frac{y^2}{2} - \frac{Hy}{3}\right) + \frac{y\tau_i}{\mu_{\rm A}}$, $\tau_i = \frac{\partial P}{\partial x}\left(\frac{H}{6\mu_{\rm A}} - \frac{2H}{3\mu_{\rm B}}\right) \Big/ \left(\frac{1}{\mu_{\rm A}} + \frac{2}{\mu_{\rm B}}\right)$, $-H\frac{\partial P}{\partial x}$, $\frac{\rho_{\rm A} D}{\mu_{\rm A}}\left(\frac{\tau_i H^2}{18} - \frac{H^3}{81}\frac{\partial P}{\partial x}\right)$
4. $\frac{8\mu\dot{m}}{\rho\pi}\left(\frac{L_1}{R_1^4} + \frac{L_2}{R_2^4}\right)$
5. $96/{\rm Re}_{D_{\rm H}}$

Due Tuesday February 25th at 11:00. Do Questions #3 and #5 only.