Fundamentals of Fluid Mechanics B
Assignment 5 — Stokes and Oseen Flow

Instructions

Write your solutions in single column format, with one statement following another vertically. Write your solutions neatly so that they are easy to read and verify. Don't write one line with two equal signs. Highlight your answers using a box. Failure to do this will result in a lower score and fewer comments on my part.

Problem #1

(a) For a constant density flow in spherical coordinates, show that a streamfunction ψ exist and that the velocity components become:

$$v_r = \frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta}$$

$$v_\theta = -\frac{1}{r \sin\theta} \frac{\partial \psi}{\partial r}$$

List all assumptions.

(b) Assuming Stokes flow, the streamfunction for flow over a sphere can be shown to be equal to:

$$\psi = \sin^2\theta \left(\frac{A}{r} + Br + Cr^2 + Dr^4 \right)$$

Impose proper boundary conditions for flow over a sphere and find A, B, C, and D.

(c) Starting from the ψ function determined in (b) and the velocity components determined in (a), derive an expression for the drag force that the sphere experiences due to viscous effects.

Problem #2

Consider flow past a sphere:

(a) Using order of magnitude analysis, determine the conditions for which the convection terms can be assumed negligible compared to the other terms in the vicinity of the sphere surface.

(b) Using order of magnitude analysis, explain why under the conditions found in (a) the convection terms can not be assumed negligible compared to the diffusion terms far away from the sphere surface.

Hint: use the general Stokes flow solution to find an order of magnitude approximation to the derivatives close to and far from the sphere.

Problem #3

The average diameter of droplets originating from expiratory activities (breathing, talking, coughing) can be taken to be 75 micrometers but with a substantial fraction having a diameter as small as 5 micrometers:

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2843952/

Let's determine how long such droplets remain in the air after being exhaled at a height of 1.6 m above the ground. Specifically, do the following:

(a) First determine analytically an expression for the velocity of the droplets and the time needed for the droplets to hit the ground assuming that the inertia terms (within Newton's law) are negligible. Do this using Stokes solution and Oseen's solution. Find the time in seconds and velocity in m/s using the Stokes and Oseen's solutions.

(b) Starting from Newton's law and neglecting aerodynamic drag (due to pressure or shear stresses), determine the time in seconds needed for the droplet to hit the ground. Do this for both Stokes and Oseen's solutions.

(c) Determine as accurately as possible (including inertia effects and aerodynamic drag) the time needed in seconds for such droplets (both 75 and 5 micrometer diameter) to reach the ground when exhaled from an individual mouth located 1.6 m above the ground. Do this for Stokes and Oseen's solutions. If an exact solution can not be found, integrate the equations numerically using a software of your choice.

(d) Validate your results found in (c) using what was found in (a) and (b). Explain your logic.

(e) Verify that the low Reynolds number assumption made in (c) is valid for this problem.

You can assume that the droplet properties are similar to the ones of water. You can take the water and air properties at ambient conditions (1 atm and 300 K) from the tables.

Answers

1. The viscous drag is less than the sum of the viscous and pressure drag listed in the tables but is function of the same terms. Note that you should obtain at one point $\tau_{r\theta} = -1.5\mu U \sin(\theta) R^3/r^4$.

3. (b) 0.5711 s; (c) Stokes: 9.658 s, 2169 s; Oseen: 11.98 s, 2169 s.

Due on Thursday March 21st at 11:00. Do all 3 problems.