Fundamentals of Fluid Mechanics B
Assignment 6 — Rayleigh and Blasius

Instructions

Write your solutions in single column format, with one statement following another vertically. Write your solutions neatly so that they are easy to read and verify. Don't write one line with two equal signs. Highlight your answers using a box. Failure to do this will result in a lower score and fewer comments on my part.

Question #1

Derive the following expression for the velocity distribution in a laminar boundary layer using a polynomial fit through the boundary conditions:

$$\frac{u}{u_\infty} = \frac{3y}{2\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Do the following:

(a) Outline all assumptions

(b) Explain the origin of each boundary condition and why such is valid for a boundary layer

(c) Prove that the above does indeed satisfy all boundary conditions.

Question #2

Recall the Rayleigh's solution to the boundary layer thickness:

$$\frac{\delta}{x} = 3.68\ \mathrm{Re}_x^{-0.5}$$

Recall that we derived in class another solution to the boundary layer thickness:

$$\frac{\delta}{x} = 4.64\ \mathrm{Re}_x^{-0.5}$$

Note that the Blasius solution is

$$\frac{\delta}{x} = 4.9\ \mathrm{Re}_x^{-0.5}$$

Explain why these solutions are different from each other.

Question #3

Starting from the x-component of the momentum equation:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+\mu\frac{\partial^2 u}{\partial z^2}$$

and from the mass conservation equation:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

When assuming that the velocity is obtained through a polynomial fit through the boundary conditions as follows:

$$\frac{u}{u_\infty}=\frac{3y}{2\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

show that the skin friction coefficient and the thickness of a laminar boundary layer correspond to:

$$C_f=0.647\cdot \mathrm{Re}_x^{-0.5} \quad \text{and} \quad \delta/x=4.64\cdot \mathrm{Re}_x^{-0.5}$$

Outline all assumptions.

Due on Tuesday April 2nd at 11:00. Do all problems.