2021 Fundamentals of Fluid Mechanics B Midterm Exam

Tuesday March 22nd 2021
11:00 — 12:15

INSTRUCTIONS

- CLOSED BOOK EXAM: NO TEXTBOOK OR CLASS NOTES OR ANY NOTES ALLOWED OTHER THAN THE FUNDAMENTALS OF FLUID MECHANICS TABLES.
- ONLY CALCULATORS WITH NO SD CARD CAPABILITIES ARE ALLOWED.
- ALL QUESTIONS HAVE EQUAL VALUE; ANSWER ALL 2 QUESTIONS.
- WRITE YOUR SOLUTIONS IN SINGLE COLUMN FORMAT, WITH ONE STATEMENT FOLLOWING ANOTHER VERTICALLY.
- WRITE YOUR SOLUTIONS NEATLY SO THAT THEY ARE EASY TO READ AND VERIFY.
- DON'T WRITE ONE LINE WITH TWO EQUAL SIGNS.
- HIGHLIGHT YOUR ANSWERS USING A BOX.
- USE A BLACK PEN OR A PENCIL TO WRITE YOUR SOLUTIONS. DO NOT USE NON-BLACK INK.

Question #1

Starting from the non-constant-density and non-constant-viscosity Navier-Stokes equations and the mass conservation transport equation, show that the constant-density and constant-viscosity y-momentum equation corresponds to:

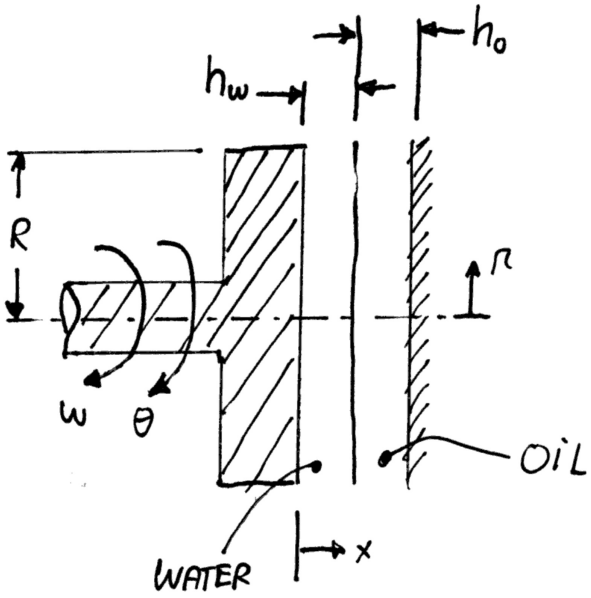
$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\frac{a}{2}\frac{\partial^2 v}{\partial x^2}+\frac{a}{2}\frac{\partial^2 v}{\partial y^2}+\frac{a}{2}\frac{\partial^2 v}{\partial z^2}$$

Also explain why

$a=2\mu$

Question #2

Consider a disc of radius R rotating with the angular velocity ω interacting with another disc at rest. The two discs are separated by a thin layer of water of thickness h_{w} followed by a thin layer of oil of thickness h_{o}, as follows:



The device is built such that the oil and the water are prevented to move out (radially) of the region between the two discs. Given the thicknesses h_{w}, h_{o}, the viscosity of the water μ_{w}, the viscosity of the oil μ_{o}, the density of the water ρ_{w}, the density of the oil ρ_{o}, the angular velocity of the left disc ω, find the velocity components within the oil and the water everywhere (at any x, r, θ). You can neglect gravity forces. Outline clearly your assumptions.