

2021 Fundamentals of Fluid Mechanics

B Midterm Exam

Tuesday March 22nd 2021
11:00 — 12:15

INSTRUCTIONS

- CLOSED BOOK EXAM: NO TEXTBOOK OR CLASS NOTES OR ANY NOTES ALLOWED OTHER THAN THE FUNDAMENTALS OF FLUID MECHANICS TABLES.
- ONLY CALCULATORS WITH NO SD CARD CAPABILITIES ARE ALLOWED.
- ALL QUESTIONS HAVE EQUAL VALUE; ANSWER ALL 2 QUESTIONS.
- WRITE YOUR SOLUTIONS IN SINGLE COLUMN FORMAT, WITH ONE STATEMENT FOLLOWING ANOTHER VERTICALLY.
- WRITE YOUR SOLUTIONS NEATLY SO THAT THEY ARE EASY TO READ AND VERIFY.
- DON'T WRITE ONE LINE WITH TWO EQUAL SIGNS.
- HIGHLIGHT YOUR ANSWERS USING A BOX.
- USE A BLACK PEN OR A PENCIL TO WRITE YOUR SOLUTIONS. DO NOT USE NON-BLACK INK.

Question #1

Starting from the non-constant-density and non-constant-viscosity Navier-Stokes equations and the mass conservation transport equation, show that the constant-density and constant-viscosity y -momentum equation corresponds to:

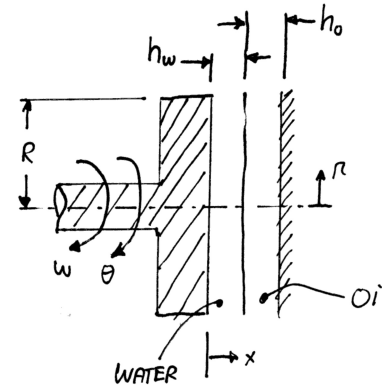
$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial P}{\partial y} + \frac{a}{2} \frac{\partial^2 v}{\partial x^2} + \frac{a}{2} \frac{\partial^2 v}{\partial y^2} + \frac{a}{2} \frac{\partial^2 v}{\partial z^2}$$

Also explain why

$$a = 2\mu$$

Question #2

Consider a disc of radius R rotating with the angular velocity ω interacting with another disc at rest. The two discs are separated by a thin layer of water of thickness h_w followed by a thin layer of oil of thickness h_o , as follows:



The device is built such that the oil and the water are prevented to move out (radially) of the region between the two discs. Given the thicknesses h_w , h_o , the viscosity of the water μ_w , the viscosity of the oil μ_o , the density of the water ρ_w , the density of the oil ρ_o , the angular velocity of the left disc ω , find the velocity components within the oil and the water everywhere (at any x , r , θ). You can neglect gravity forces. Outline clearly your assumptions.