Fundamentals of Fluid Mechanics B
Assignment 9 — Vorticity Dynamics

Instructions

Write your solutions in single column format, with one statement following another vertically. Write your solutions neatly so that they are easy to read and verify. Don't write one line with two equal signs. Highlight your answers using a box. Failure to do this will result in a lower score and fewer comments on my part.

Question #1

Derive Kelvin's circulation theorem. Specifically, do the following:

(a) Prove that

$$\frac{d(d\xi_i)}{dt} = dv_i$$

(b) Prove that

$$\oint_C d\left(\frac{v_i^2}{2}\right) = 0$$

(c) Prove that

$$\oint_C \frac{\partial P}{\partial x_i} d\xi_i = 0 \quad \text{or} \quad \sum_i \oint_C \frac{\partial P}{\partial x_i} d\xi_i = 0$$

(d) Using (a), (b), (c), and starting from the definition of the circulation:

$$\Gamma \equiv \oint_C \vec{v} \cdot d\vec{\xi}$$

Prove that the following is correct:

$$\frac{d\Gamma}{dt} = \sum_{i=1}^{3} \oint_C \left(\frac{\mu}{\rho}(\vec{\nabla} \cdot \vec{\nabla}) v_i\right) d\xi_i$$

Question #2

Starting from the velocity distribution expression obtained through a polynomial fit through the boundary conditions, calculate all components of the vorticity vector within a constant pressure boundary layer over a flat plate at a certain x distance from the leading edge. Plot the vorticity as a function of the y coordinate. Are the results in accordance with the rates of vorticity input to the flow at the wall seen in class?

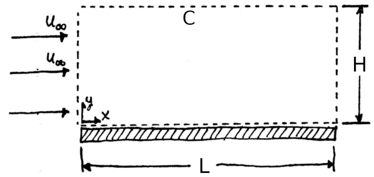
$$\frac{\partial \vec{\omega}_z}{\partial y} = -\frac{1}{\mu}\frac{\partial P}{\partial x}$$

$$\frac{\partial \vec{\omega}_x}{\partial y} = \frac{1}{\mu}\frac{\partial P}{\partial z}$$

Why or why not? Explain.

Question #3

Consider a fluid flowing on top of a flat plate as follows:



Starting from the Kelvin's circulation theorem:

$$\frac{d\Gamma}{dt} = \sum_{i=1}^{3} \oint_C \left(\frac{\mu}{\rho}(\vec{\nabla} \cdot \vec{\nabla}) v_i\right) d\xi_i$$

Calculate the rate of change of the circulation $\frac{d\Gamma}{dt}$ within the contour C for $H = 2\delta_{x=L}$ with δ being the thickness of the boundary layer. Express $d\Gamma/dt$ as a function of the free stream fluid properties u_∞, ρ, and μ and the dimensions L and H, and simplify the expression as much as possible. Hint: the rate of change of Γ is not zero.

Answers

3. $-\frac{41.76 u_\infty^2}{\mathrm{Re}_H \sqrt{\mathrm{Re}_L}}$

Due on Tuesday May 4th at 11:00. Do all problems.